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Some Lessons from Modeling Ground Water Flow in the Metropolitan Area of Milano at Different Scales

Alcuni insegnamenti dalla modellazione del flusso idrico sotterraneo nell'area metropolitana milanese a diverse scale

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ABSTRACT - Flow and transport in porous media can be modelled at very different scales: from the molecular scale (length scale or model resolution, *l*, approximately equal to 10^{-10} m), to the microscopic scale (pore-grain scale, $l \approx 10^{-5} \div 10^{-3}$ m), the macroscopic scale (representative elementary volume, $l \approx 10^{-3} \div 10^{-1}$ m), the megascopic scale used for local field problems ($l \approx 1 \div 10$ m) and the gigascopic scale at which regional aquifer systems are studied ($l \approx 10^2 \div 10^3$ m).

The link between the megascopic and gigascopic scales is important for practical applications and is discussed in this communication with reference to the flow models developed by our research group in the last years for the aquifer system of the metropolitan area of Milano. In fact we have developed several nested models, which consider areas varying between 20 and 700 km^2 and with grid spacing varying between 100 and 1500 m.

Models developed and applied at different scales can reproduce different features and require different data for the calibration and validation. This communication will give practical examples about the following problems: the adequacy of data (data accuracy; spatial and temporal density of data point; data effectiveness in representing phenomena at the appropriate scale); model calibration with multiple sets of data; model validation through the "forecasting" of natural undisturbed conditions of the aquifer system.

KEY WORDS: Ground Water, Aquifer Systems, Mathematical Modeling, Scaling, Calibration and Validation.

RIASSUNTO - I fenomeni di flusso e trasporto nei mezzi porosi possono essere modellati su scale molto differenti: dalla scala molecolare, che corrisponde a lunghezze di scala o risoluzione dei modelli, *l*, dell'ordine di 10^{-10} m, alla scala microscopica (scala dei pori e dei granuli, $l \approx 10^{-5} \div 10^{-3}$ m), alla scala macroscopica (a cui si può definire il volume elementare rappresentativo, $l \approx 10^{-3} \div 10^{-1}$ m), alla scala megascopica usata per problemi su aree relativamente ristrette ($l \approx 1 \div 10$ m) e infine alla scala gigascopica alla quale vengono studiati i sistemi aquiferi regionali ($l \approx 10^2 \div 10^3$ m).

Il legame tra le scale megascopica e gigascopica è importante per le applicazioni pratiche e viene discusso in questa comunicazione facendo riferimento ai modelli di flusso sviluppati per l'area metropolitana Milanese. Infatti il nostro gruppo di ricerca ha sviluppato diversi modelli innestati, che considerano superfici variabili tra 20 e 700 km^2 e con spaziatura delle griglie di discretizzazione variabile tra 100 e 1500 m.

I modelli sviluppati e applicati a scale differenti possono riprodurre processi diversi e richiedono dati diversi per la calibrazione e la validazione. In questa comunicazione vengono dati alcuni esempi concreti su problemi come: l'adeguatezza dei dati (accuratezza dei dati, densità spazio-temporale dei punti di campionamento); la calibrazione del modello con più insiemi di dati; la validazione del modello attraverso la "previsione" delle condizioni naturali del sistema acquifero.

PAROLE CHIAVE: Acque sotterranee, Sistemi acquiferi, Modellazione matematica, Variazioni di scala, Calibrazione e Validazione.

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1. - INTRODUCTION

Flow and transport in porous media can be modelled at several different scales, which correspond to different physical, chemical or geological objects and processes and to different purposes. A tentative ranking of scales is given in table 1 (see also DAGAN, 2000; LUNATI *et alii*, 2001), where the scales are ordered according to the scale length, *l*, which can be identified with the model resolution or, in other words, the smallest length at which the processes can be described. The term "model resolution" is used because it can be considered equivalent to the grid spacing in a numerical flow or transport model.

The finest scale is the molecular scale ($l \cong 10^{-10}$ m), at which the important interactions are molecular forces, acting among the molecules constituting the fluid (liquid or gas) and solid phases. When the physical quantities are averaged over statistically significant volumes, that include a large number of molecules, but are nevertheless small as to provide a point value, we can move to the microscopic scale (pore-grain scale, $10^{-6} \leq l \leq 10^{-3}$ m), which is the scale at which the classical dynamics of fluids is developed. At this scale it is significant to define fluid pressure, solid stress, temperature, etc.

However, from the point of view of the study of flow in porous geological formations, this is not the most appropriate scale and a further step has to be done, toward the macroscopic scale (representative elementary volume, $10^{-3} \leq l \leq 10^{-1}$ m). At this scale it is possible to define quantities representative of the physics of the porous medium, i.e. taking into account the presence of several phases (fluid and solid phases) and possibly several constituents within each phase. This is the scale relevant for laboratory experiments on samples, unsaturated flow and field tests involving small volumes.

Local field problems ($1 \leq l \leq 10$ m) can be studied at the megascopic scale; this is the case for pumping tests or 3D models of flow and transport for the characterization and remediation of contaminated sites.

Finally, regional aquifer systems are studied at the gigascopic scale ($10^2 \leq l \leq 10^3$ m), to develop engines of decision support tools for resource management and regional planning. This is usually done introducing the hypothesis of quasi-3D flow.

The link between macro-, mega- and possibly giga-scopic scales is sometimes performed with studies on aquifer analogues, i.e., outcropping sediments which are representative of the burden water reservoirs (see, e.g., JUSSEL *et alii,* 1994; RITZI *et alii,* 1995; WITTAKER & TEUTSCH, 1996; HUGGENBERGER & AIGNER, 1999; BERSEZIO *et alii*, 1999; ANDERSON *et alii,* 1999; ZAPPA *et alii*, 2006). The heterogeneous conductivity field, which is mapped at the macroscopic scale on the basis of the facies distribution, is upscaled to find equivalent parameters of the porous medium at the macro- or giga-scopic scale (for thorough reviews on upscaling see WEN & GÓMEZ-HERNÁNDEZ, 1996; RENARD & MARSILY, 1997; CUSHMAN *et alii*, 2002).

Within this framework, the aim of this presentation is to show the importance of the scales in practical applications and how the choice of the model scale length can affect not only modeling aspects, but also data collection. In this paper this topic is discussed with some examples taken from the models developed by our research group for the aquifer system of the metropolitan area of Milano (Italy) (fig. 1).

For a theoretical discussion of the problem of the relationships between measurement, scales and model parameters, which is beyond the goal of this paper, the reader is referred to (CUSHMAN, 1986; BECKIE, 1996); CUSHMAN *et alii,* 2002).

The examples of models for the aquifer system of the metropolitan area of Milano provide also some interesting remarks about model calibration and validation.

2. - MODELS DEVELOPED FOR THE METROPOLITAN AREA OF MILANO

The city of Milano (Northern Italy) draws water for both domestic and industrial purposes from ground water resources located beneath the urban area. The rate of ground water abstraction has been varying during the XX Century, depending upon the number of inhabitants and the development of industrial activities. The ground water abstraction raised to a maximum of about $700 \cdot 10^6$ m^3/year in the middle 1970s and has successively decreased to a value of about $400 \cdot 10^6$ m^3/year at present days. This caused time variations of the depth of the water table below the ground surface and in turn some emergencies: the two most prominent episodes correspond to the middle 1970s, when the water table in the city centre was about 30 m below the undisturbed natural conditions, and to the last decade of the XX Century, when the water table raised at a rate of approximately 1 m/year and caused infiltrations in - sometimes, flooding of - deep constructions (garages, building foundations, the under-

Tab. 1 - *Scale lengths at which ground water flow can be studied.*
- Lunghezze di scala a cui può essere studiato il flusso idrico sotterraneo.

Scale length or model resolution (m)	10^{-10}	$10^{-6}\div10^{-3}$	$10^{-3}\div10^{-1}$	$1\div10$	$10^{2}\div10^{3}$
Scale	Molecular	Microscopic	Macroscopic	Megascopic	Gigascopic

ground railways, etc.).

The largest variations have been observed in the central and northern areas of the city, which have been subject to a strong development during the XX Century, whereas south of the city the aquifer system is largely controlled by the hydrographic network, which keeps the water table quite shallow and permits small oscillations with time.

In the study area ground water flows through a sequence of continental and transitional deposits overlying an early Pleistocene marine clay bedrock. The hydrostratigraphic framework has been recently revised by REGIONE LOMBARDIA & ENI DIVISIONE AGIP (2002) and the reader is referred to that work for a thorough description of the regional hydrostratigraphy. Here it is sufficient to recall few results.

The aquifer system has been subdivided into four aquifer groups, labelled with the letters A, B, C and D from top to bottom, as represented in table 2. The uppermost aquifer group A consists

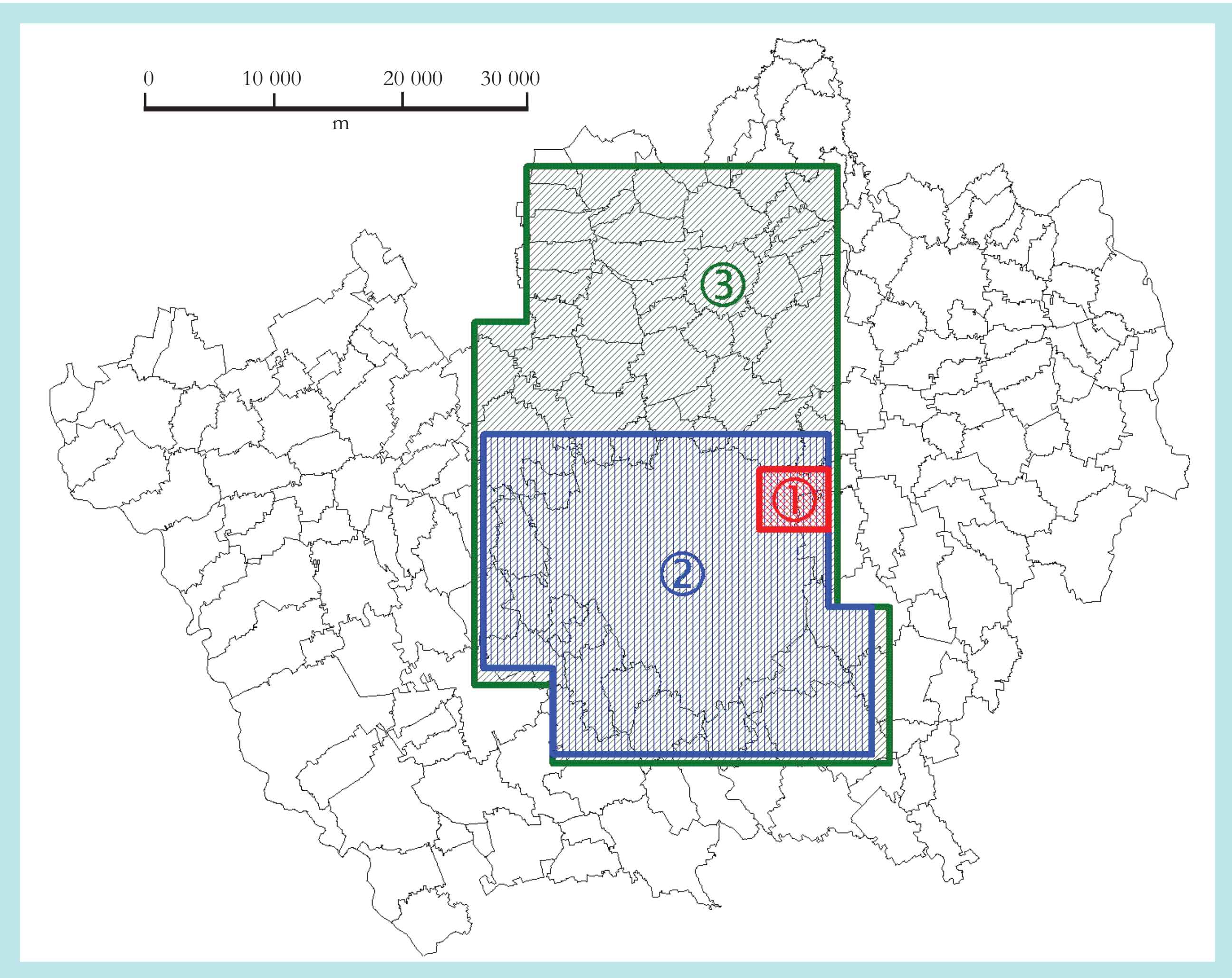

Fig. 1 - Map of the Province of Milano, with municipality borders and the study areas of the three models described in this paper: (1) modLambro, (2) ModMil and (3) ProvMI.
- *Mappa della Provincia di Milano con i confini comunali e le aree analizzate con i tre modelli descritti in questo articolo: (1) modLambro, (2) ModMil and (3) ProvMI.*

of gravel and sand sediments of alluvial and glacial origin, with a thickness of few tens of meters. The underlying aquifer group B is composed of glacial and fluvio-glacial sands and gravels with a thickness ranging from 50 to 90 m. These aquifer groups are clearly separated in the area of the city of Milano, whereas the distinction is less apparent moving toward North. Moreover, conglomerate layers have been observed at distinct depth ranges, with a limited extension in the area of the city, but with increasing thickness and lateral continuity in the northern part of the study area. The aquifer groups A and B together correspond to the so called "traditional aquifer", i.e. the aquifer which has been mainly exploited. In recent years, due to the problems mentioned at the beginning of this section, some wells have been drilled and screened in the aquifer group C, consisting of sediments deposited in a deltaic or marine environment, with a predominance of silt and clay in which sandy aquifers (commonly named "deep aquifers") are embedded.

Tab. 2 - *Hydrostratigraphic units and aquifer groups (modified after* REGIONE LOMBARDIA & *ENI* DIVISIONE *AGIP, 2002).*
-Unità idrostratigrafiche e gruppi acquiferi (modificato da REGIONE LOMBARDIA & ENI DIVISIONE AGIP, 2002).

Chronostratigraphic scale (10^6 year)		Traditional hydrostratigraphic units		Aquifer groups
0.01	Olocene	*Traditional aquifer*	Gravel-sandy unit	A
0.12	Upper Pleistocene			
	Middle Pleistocene		Gravel-sandy-silty unit	
			Conglomerate and basal sandstones unit	B
0.89		*Deep aquifers*	Sandy-silty unit (continental and transitional facies)	C
1.73	Lower Pleistocene		Silty-clay unit (marine facies)	D
	Upper Pliocene			

In this paper three models are considered (see table 3 and figure 1). The specific characteristics of each model are listed in table 3, whereas they share the following features: they are based on the quasi-3D approximation (horizontal flow in the aquifers and vertical flow in the aquitards), apply conservative finite-differences schemes for regular grids with square cells in the horizontal plane and are implemented with proprietary computer codes.

The first model to be developed is ModMil (GIUDICI *et alii*, 2000), which simulates ground water flow in the traditional aquifer over an area of about 400 km^2 and includes the municipality of Milano; in the area more than 600 public wells and more than 400 private wells are drilled.

Then modLambro (ROMANO, 2001; ROMANO *et alii*, 2002) has been developed to model an area of about 20 km^2, located at the eastern border of the city, largely occupied by a park area, crossed by the Lambro river and including four pumping stations of the municipal Water Works; modLambro considers both the traditional and the deep aquifers (thickness of almost 200 m) and has a grid spacing equal to 100 m. Since the physical boundaries of the aquifer system are far from the modelled area, the results of ModMil

Tab. 3 - *List of some mathematical models of ground water flow for the metropolitan area of milano.*
-Elenco di alcuni modelli matematici del flusso idrico sotterraneo nell'area metropolitana Milanese.

Model	Modelled area	Extension of the modelled area (km^2)	Modelled aquifers	Grid spacing (m)
modLambro	A small area of the city of Milano	20	Traditional and deep aquifers	100
ModMil	City of Milano and some neighbouring municipalities	400	Traditional aquifer	500
ProvMI	One third of the province of Milano	700	Traditional and deep aquifers	1500

are used to fix the boundary conditions of modLambro. Both Dirichlet (prescribed heads) and Neumann (prescribed fluxes) boundary conditions are considered and the results show significant differences for the two alternatives. Neumann boundary conditions, assigned on the basis of the results of ModMil, appear to be the preferable choice, because they leave more "freedom" to the nested model, in particular if some wells are abstracting water close to the border.

The third model, ProvMI, has been developed as an extension of ModMil toward North, up to the hills formed by the moraines deposited by alpine glaciers. It models the traditional and deep aquifers and has a grid spacing equal to 1500 m.

3. - ARE AVAILABLE DATA USEFUL FOR MODEL DEVELOPMENT?

The answer to the question posed in the title of this section should be negative, in general. In fact, most of the available data are collected with goals very different from the development and application of a flow model, but are usually related to a general monitoring of the system.

Most of the available data come from existing production wells, some of which have been drilled several tens of years ago. The location of the wells is often known with good precision, but their topographic level is rarely measured with precision and is often deduced by topographic maps. The abstraction rate is rarely measured at each well, but is estimated indirectly or measured for a cluster of wells. Construction data are also very important: the well depth, the depth and thickness of the screened intervals, the state of the screens, the efficiency of the drain to maintain the lateral continuity of aquitards, etc. These data are often very uncertain, above all for ancient and private wells. Moreover also the stratigraphic logs are often very uncertain both with respect to the description of the sediments encountered during the drilling and the depth of the discontinuity between different lithological levels.

Well location is not chosen on the basis of scientific considerations, but is driven by practical reasons; moreover they are numerous in areas of high conductivity, where large abstraction rates can be obtained with a small piezometric drawdown, so that the information on less permeable layers, which are very important for aquifer protection, is often scarce.

Measurements of piezometric head and of ground water quality are often collected in existing production wells. This poses a series of questions on the informative value of these data. Piezometric heads are often measured under dynamic conditions, while the well is pumping, or in so called semi-dynamic conditions, i.e. turning off the pump and waiting for some time (about 30 minutes) before taking the measurements. What do these measurements represent? Are the measured values significant to study the behaviour of the aquifer system or are they corrupted by the well characteristics?

A problem that is often faced when quasi-3D or, even worst, 3D flow models are applied is the uncertainty on the vertical component of the hydraulic gradients. In fact, if the characteristics of the wells are not known (in particular, the depth and thickness of the screened intervals), it is not possible to assign the measured value to a specific point in space. This prevents a precise evaluation of the vertical exchanges between aquifers, which would require the implementation of bundles of piezometers screened at different depths and devices to measure piezometric head with high precision.

It is important to stress that not only the spatial density of data points but also the measurement accuracy should increase for decreasing scale length or model resolution, as it is shown by a simple analysis of the order of magnitude of the physical quantities involved. Let us assume for simplicity an hydraulic gradient of about 0.005; when $l \approx 1000$ m, i.e. for a regional flow model, the variation of piezometric head over a distance l is about 5 m, whereas if $l \approx 10$ m, i.e. for a local flow model, the expected variations of piezometric head are as small as 0.05 m and require a good measurement accuracy not only of the water level in the piezometer, but also of the reference level of the piezometer.

Data about existing wells are also fundamental for the realization of the conceptual model. In fact, the stratigraphic logs obtained during the drilling are often the basis for the geological reconstruction of the subsoil, for the hydrostratigraphic studies and for the correlation of aquifers and aquitards.

The three models mentioned in the previous section (ModMil, modLambro and ProvMI) show the importance of the scale length on the identification of the hydrogeological scheme: they are based on different hydrostratigraphic schemes, as a result of different data sets and different scales.

In fact the grid spacing of ModMil (500 m) has been chosen in such a way that each cell contains at most one pumping station of the municipal Water Works. Then for each pumping station a local hydrogeological scheme has been obtained from the collection of the stratigraphic logs of the wells of the municipal Water Works processed and published by AIROLDI & CASATI (1989). From this data set it was possible to evidence those silt and clay layers which appear continuous over the area of the pumping station, whose linear extension is close to l; the local hydrogeological schemes have successively been correlated, in order to determine those aquitards that can be considered continuous throughout most of the modelled domain. This procedure led to the recognition of a layered structure of the traditional aquifer consisting of a phreatic and three leaky confined aquifers.

For ProvMI the scale length was 1500 m, and the grid has been chosen in such a way that, in the part of the modelled area which is common also to ModMil, each node of ProvMI is also a node of ModMil. In order to facilitate the future development of the model, the data set used for the identification of the hydrogeological scheme was a collection of stratigraphic logs from the huge data base of the Province of Milano, which includes public and private wells. In particular several sections have been chosen and a well for each cell along these sections has been extracted. The choice, albeit arbitrary, was based on some criteria: public wells have been preferred to private wells, because the quality of the stratigraphic logs is often better; deep wells have been preferred in order to get as much information on the deep aquifers as possible; wells close to the node of the cell have been preferred in order to avoid geometrical artefacts during the section drawing and correlation. Since the scale length is greater by three times than that of ModMil, only one aquitard appears to be continuous throughout the whole modelled area within the traditional aquifer, which is then separated into a phreatic and a semiconfined aquifer.

Finally, modLambro has the smallest scale length (100 m) and the hydrostratigraphic scheme has been identified using the stratigraphic logs of the deep wells located in the modelled area.

Among the input data for the flow models the following source terms have to be estimated and assigned (GIUDICI *et alii*, 2001):

1. abstraction from public wells for domestic use;
2. abstraction from private wells for industrial or agricultural use;
3. abstraction from topographic sources;
4. recharge from rain infiltration;
5. recharge from infiltration of water used for irrigation;
6. recharge from losses from the aqueduct and sewage networks;
7. aquifer drainage or recharge from rivers and natural or artificial channels.

As mentioned at the beginning of this section, the location of public wells is in general known with sufficient precision, and estimates of the water abstraction rates at item 1 are available, even if the flux rates are sometimes obtained indirectly from the consumption of electrical energy for water abstraction. Moreover the depth at which water is drawn from the aquifer is often unknown, because the wells have multiple screened intervals.

More uncertain is the presence and water abstraction of private wells (item 2). These data are often based on the declarations of the well owners, who usually provide incomplete or inaccurate data. Some estimates of water abstraction from private wells can be obtained from empirical relations relating water consumption and the amount of the industrial or agricultural production, taking into account the average need of water for specific productive categories.

Some data for item 3 are also available, see, e.g., PROVINCIA DI MILANO (1975) for the metropolitan area of Milano.

Instead, estimates of location and strength of items 4 and 5 of the previous list are uncertain since they depend upon a large number of variables: atmosphere parameters (precipitation, solar radiation, temperature, relative humidity, wind velocity, etc.), vegetation parameters (height of the vegetation, root depth, etc.), soil parameters (temperature and heat gradient, water content, characteristic curve and conductivity function). Estimates of evapotranspiration are often obtained with empirical equations, among which the most popular choices are Thornthwaite's and Turc's equations (MARSILY, 1986), which require the average temperature of a given time period. More complex approaches are based on the energy and mass balance at the soil and lead to the Penman's formula for evaporation and Bowen Ratio's formula (EAGLESON, 2003) or to SVAT models, that take into account the transport processes and energy and mass exchanges among soil, vegetation and atmosphere. However, these models require a lot of parameters which are seldom measured or known. Further difficulties arise from modeling the water flow through the unsaturated zone, which is very heterogeneous; the common approximation of the transmission zone, which connects the root zone with the water table, as a unique reservoir could introduce strong simplifications, not justified from the physical point of view, due to the non-linearity of the flow equation (ROMANO, 2005).

Similar problems are encountered when one tries to estimate the aquifer recharge due to losses from buried pipes of the aqueduct and sewage networks (item 6) or the exchange fluxes between the phreatic aquifer and the network of natural rivers and streams and artificial channels.

It is worthwhile to stress the importance of comparing piezometric heads with source terms. In fact, it is common, even among professionals, to plot contour maps of piezometric head using data coming from measurements performed during a given period of time: for instance piezometric heads collected at several tens of wells during a survey which lasts few days. As an alternative, contour plots of piezometric heads are sometimes obtained with time-averaged values, which are considered to be representative of the average annual flow; in particular, the average of all the measurements recorded at a position during a year (with a typical sampling frequency of one sample per month or per season) is used.

When the results are compared with location and strength of the source terms, especially of ground water abstraction, some inconsistencies are sometimes apparent, e.g. the location of depression cones different from the location of the principal production well fields, and prevent any mathematical model from being able to reproduce that situation; an example is given (VASSENA, 2004) for the aquifer system of the alluvial fan of the Reno River, close to the city of Bologna (Italy).

Another very important point related to the scale effects is the determination of hydraulic conductivity and transmissivity, or more generally the hydrodynamic physical parameters of geological formations. In fact heterogeneity appears at different scales, from the microscopic to the gigascopic scales mentioned in the introduction. It is important to notice that the results of laboratory tests on samples are representative at the macroscopic scale, whereas the results of field tests, as, e.g., pumping tests, are representative of greater scale lengths. The results of the models under consideration show that the available data can be used to identify hydrostratigraphic units, but it is difficult to recognize heterogeneity of the sediments within these units at the mega- and gigascopic scales. In other words, model results fit satisfactorily with observations, even if permeable sediments belonging to different hydrostratigraphic units are considered homogeneous and isotropic; on the other hand the available data (in particular piezometric heads and source terms) are affected by uncertainties that prevent the confident determination of spatial heterogeneity of the aquifers' sediments.

In principle, the quantification of uncertainties on the input data and on the outcomes of models can be attained with stochastic techniques, which can be useful if a large set of data are available. The research in stochastic ground water hydrology has advanced very much in the last decades, but these results have received limited application to real case studies so far (DAGAN, 2002). In particular, kriging is widely used to interpolate data and stochastic simulations are applied to reconstruct the spatial distribution of hydrogeological materials. However the stochastic methods have not yet found full application by practitioners and professionals probably because of their mathematical and conceptual complexity and for the lack of commercial user-friendly software.

4. - MODEL CALIBRATION WITH MULTIPLE SETS OF DATA

A mathematical model can be considered as a simulation tool to compute the spatial (and, possibly, temporal) distribution of the potential (piezometric head) describing the state of the system from a set of physical parameters which characterise the natural system (hydraulic conductivity, source terms, boundary conditions). Among the crucial aspects of the model development and application (HASSANIZADEH & CARRERA, 1992; GIUDICI, 1999, 2001), calibration and validation should be mentioned. The objective of model calibration is the identification of the "optimal" values of the model parameters, which give the best (or at least a satisfactory) fit between model results and observations. In principle one cannot guarantee that a calibrated model will forecast the behaviour of the natural system for any flow condition; the optimal values of model parameters for a given flow condition could not be the optimal values for a different flow situation (KONIKOW, 1986). Therefore, model validation, also called "post-audit" (KONIKOW, 1986; ANDERSON & WOESSNER, 1991; HASSANIZADEH & CARRERA, 1992; KONIKOW & BREDEHOFT, 1992; GIUDICI, 2003), aims to check whether the calibrated model is valid to simulate several different flow conditions, without any further tuning of the model parameters.

Model calibration can be obtained through the solution of an inverse problem, as described in some seminal papers and several review papers and books (e.g., NELSON, 1960, 1961; EMSELLEM & MARSILY, 1971; SCARASCIA & PONZINI, 1972;

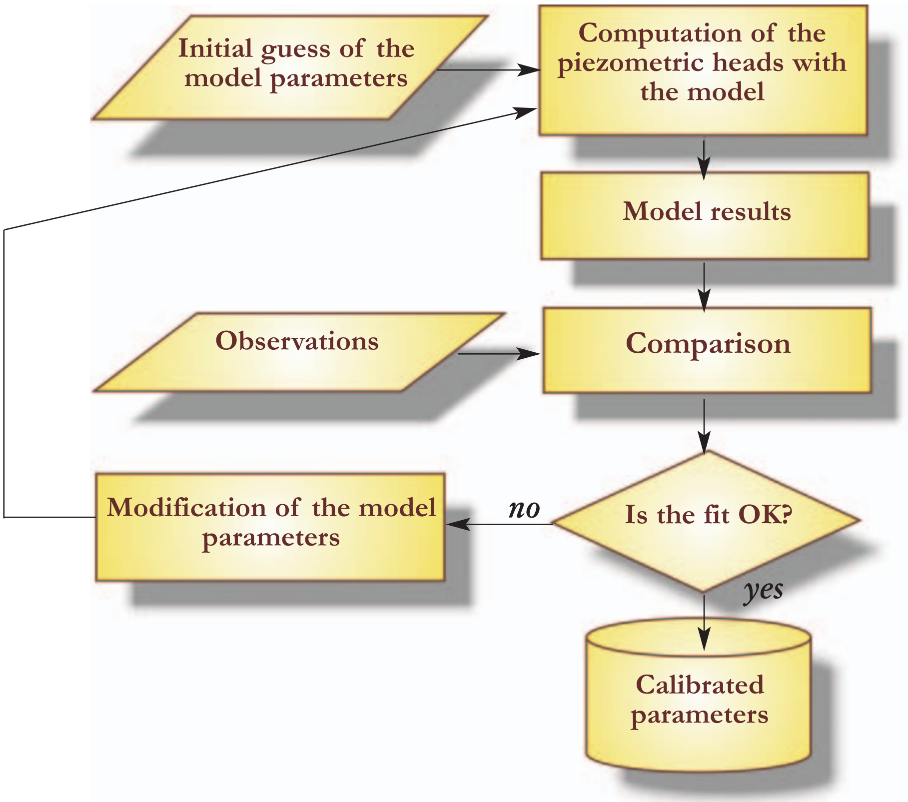

Fig. 2 - *Scheme of the procedure of model calibration with a "trial and error" process.*
- Schema della procedura di calibrazione del modello con un approccio *"trial-and-error"*.

NEUMAN, 1973; NEUMAN & YAKOWITZ, 1979; YEH, 1986; CARRERA, 1988; GINN & CUSHMAN, 1990; HASSANIZADEH & CARRERA, 1992; SUN, 1994; MCLAUGHLIN & TOWNLEY, 1996; HILL, 1998; ZIMMERMAN *et alii*, 1998; GIUDICI, 1999, 2001, 2003); it is often performed with a "trial and error" procedure, sketched in figure 2. The comparison between model results and observations is often based on different definitions of model errors, e.g., the average absolute error (l^1-norm in the mathematical language) or the average squared error (l^2-norm in the mathematical language):

$$e_1 = \frac{1}{N}\sum_{i=1}^{N}\left|h^{obs}{}_i - h^{pre}{}_i(p)\right| \qquad \text{and}$$

$$e_2 = \frac{1}{N}\sum_{i=1}^{N}\left[h^{obs}{}_i - h^{pre}{}_i(p)\right]^2$$

where N is the number of available observations, $h^{obs}{}_i$ and $h^{pre}{}_i$ (p), i =1,... N, are respectively the observed piezometric heads and the piezometric heads computed with the model for the values of the model parameters listed in the array p.

It has been proved theoretically the importance of using several independent data sets (for both stationary and transient conditions) to reduce uncertainty in the model calibration (SAGAR *et alii*, 1975; CARRERA & NEUMAN, 1986a, 1986b, 1986c; GINN *et alii*, 1990; SNODGRASS & KITANIDIS, 1998; PARRAVICINI *et alii*, 1995; GIUDICI *et alii*, 1995; VÁZQUEZ GONZÁLEZ *et alii*, 1997). This result is largely independent from the used inversion method, both within a deterministic and a stochastic framework. From a conceptual point of view, the use of multiple data sets is important, since it permits to perform a procedure similar to a tomographic imaging. In fact, multiple data sets provide independent information if the flow directions differ from each other. Therefore, the results of model calibration performed with data sets corresponding to different flow situations are more confident and, as a consequence, the use of several independent sets of data improves the reliability of model forecasting.

This remark is clearly demonstrated from the results of ModMil. In fact for that model the

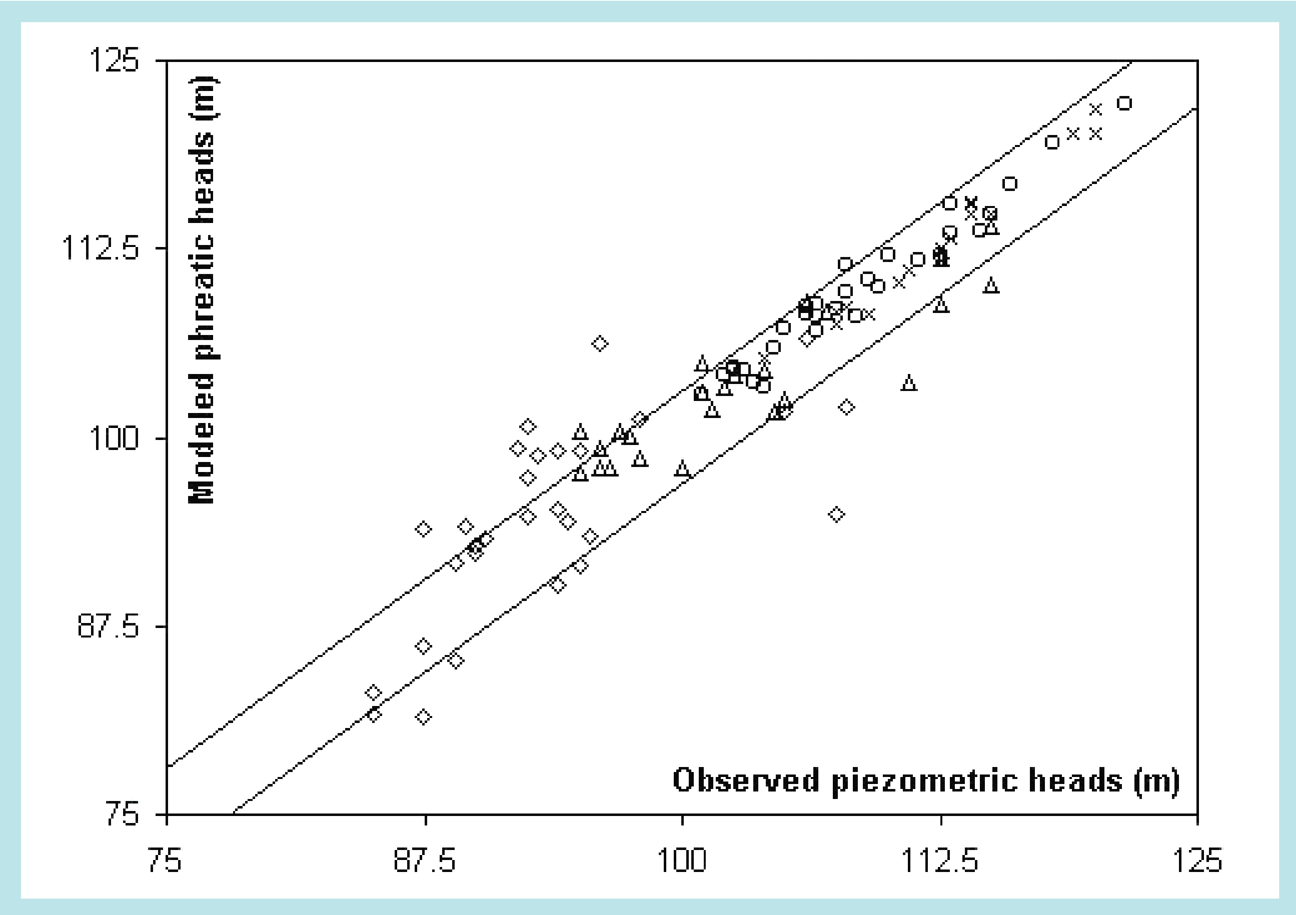

Fig. 3 - *Comparison between modelled phreatic heads and observed piezometric heads for ModMil. The symbols show data corresponding to different years: circles (1950), diamonds (1974) and triangles (1982) refer to the data used for calibration; crosses refer to the data used for validation (1997). The area delimited by the continuous straight lines includes the data with absolute difference between modelled and observed heads smaller than 2.5 m.*

- Confronto tra le altezze freatiche modellate e le altezze piezometriche osservate per ModMil. I simboli mostrano dati corrispondenti a diversi anni: cerchi (1950), diamanti (1974) e triangoli (1982) si riferiscono ai dati usati per la calibrazione; le croci si riferiscono ai dati usati per la validazione (1997). L'area delimitata dalle linee rette continue include i dati con differenze tra le altezze osservate e modellate il cui valore assoluto è inferiore a 2.5 m.

available data were used in a two-step process. Some of the available data were used for model calibration; the remaining data were used to validate the model, i.e. to check if the model predictions for flow conditions different from those used in the calibration are still close to the observations, without any adjustment of the model parameters.

The results obtained for ModMil are represented in figure 3, where the modelled piezometric heads are plotted against the observed piezometric heads at the same positions. The difference between modelled and observed heads for 1997, the year used for validation, is about the same as for the three years used for the calibration: 1950 corresponds to the first period of intensive ground water abstraction; 1974 corresponds to the maximum intensive aquifer exploitation; 1982 corresponds to a first rising of the water table. In other words, these comparisons show that the expected discrepancy between model forecast and real observations is of the same order of magnitude as that obtained during the calibration stage and this is an important information if the model forecast have to be used for the management of water resources.

5. - MODEL VALIDATION THROUGH THE "FORECASTING" OF NATURAL UNDISTURBED CONDITION OF THE AQUIFER SYSTEM

Data available for the application of a ground water flow model usually refer to recent periods, during which an extensive aquifer exploitation occurred. It is not always possible to collect enough data to apply the two-stage procedure discussed in the previous section (use part of the data for calibration and the remaining data for validation). However it is often possible to get information about the undisturbed condition of the aquifer system. In fact, historical data could be available or at least some constraints can be imposed and must be satisfied by the model.

For instance, historical data show that until the late XIX century the water table in the city centre of Milano was a few meter below the ground, where an extensive channel network was present. Then, after the foundation of the municipal Water Works in 1889, the aquifer exploitation has started and the water table lowered in the city centre. Nevertheless, several topographic springs were still present and active in the neighbourhoods of the city until middle 1960s; they were confined mainly along two strips, north and south of the city, where the boundaries of ModMil have been chosen. Thus, it was possible to assign physically based Dirichlet boundary conditions to ModMil, at least if the ground water flow is modelled until 1960s. The springs located north of the city are nowadays expired, whereas south of the city, the phreatic level is strongly controlled by the interactions between the aquifer system and the natural and artificial drainage network, so that the oscillations of the water table are small and several springs are still present.

This hydrogeological situation permits to fix boundary conditions for ModMil, but is useful to validate ProvMI. In fact the latter model refers to a larger area, so that the line of springs north of the city cannot be introduced as a boundary condition for undisturbed flow conditions, but has to be reproduced by the model itself. In particular when ProvMI runs without abstraction fluxes, the following constraints have to be satisfied: the modelled water table at the line of the springs has to be at most a couple of meters below the ground surface, it must not be over the ground surface in the city centre, because there is no historical evidence of flooded area, and it must be in accordance with the observations done at the beginning of the XX century and reproduced in technical reports for the design of the sewage network (MUNICIPIO DI MILANO - UFFICIO TECNICO, 1911). In order to satisfy the above mentioned constraint it has been necessary to introduce abstraction rates from the cells of ProvMI corresponding to the area of the topographic springs north of the city: the values of this source term is in good agreement with the available data (PROVINCIA DI MILANO, 1975).

These data are often available in practical applications, especially in urbanized areas, as most of the Po plain, which were subject to drainage works in historical periods (see also VASSENA, 2004).

6. - CONCLUSIONS

We can finally summarise some of the most important suggestions that came out from the development of a series of ground water flow models for the metropolitan area of Milano. These lessons are nevertheless of general validity.

First of all modellers often face great difficulties because the available data are not adequate for the development, calibration and validation of a numerical flow model. This is of paramount importance, because any model cannot be used as a forecasting tool in a confident way if it is not well calibrated and validated by comparing model outcomes and field observations. In other words we can state that no modeling exercise can ignore quality and density of the existing data (see also the discussion in GIUDICI, 2001).

However all the available data should be exploited at the maximum of their possibility. In particular the following guidelines could be provided:

1. use part of the available data set for calibration and the remaining data for model validation;
2. use as many data corresponding to different flow conditions as possible for the model calibration;
3. use as many information as possible, e.g., historical and geomorphological data, in order to reproduce the natural undisturbed behaviour and to check whether the conceptual model and the mathematical model predict the natural condition of the aquifer system.

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