Bolam: dinamica e numerica

Variabili indipendenti:

$t, \ \varphi\ (\theta), \ \ \lambda, \ \ z$

Variabili dipendenti:

u, v, w, T, P, ρ

$q, q_{CW}, q_{CI}, q_{PW}, q_{PI1}, q_{PI2}$

$$P = \rho \, R_d \, T_V$$

Equazione di stato

$$T_V = T \left(1 + (\frac{1}{\varepsilon} - 1) \, q - \sum_k q_k \right)$$

Temperatura virtuale

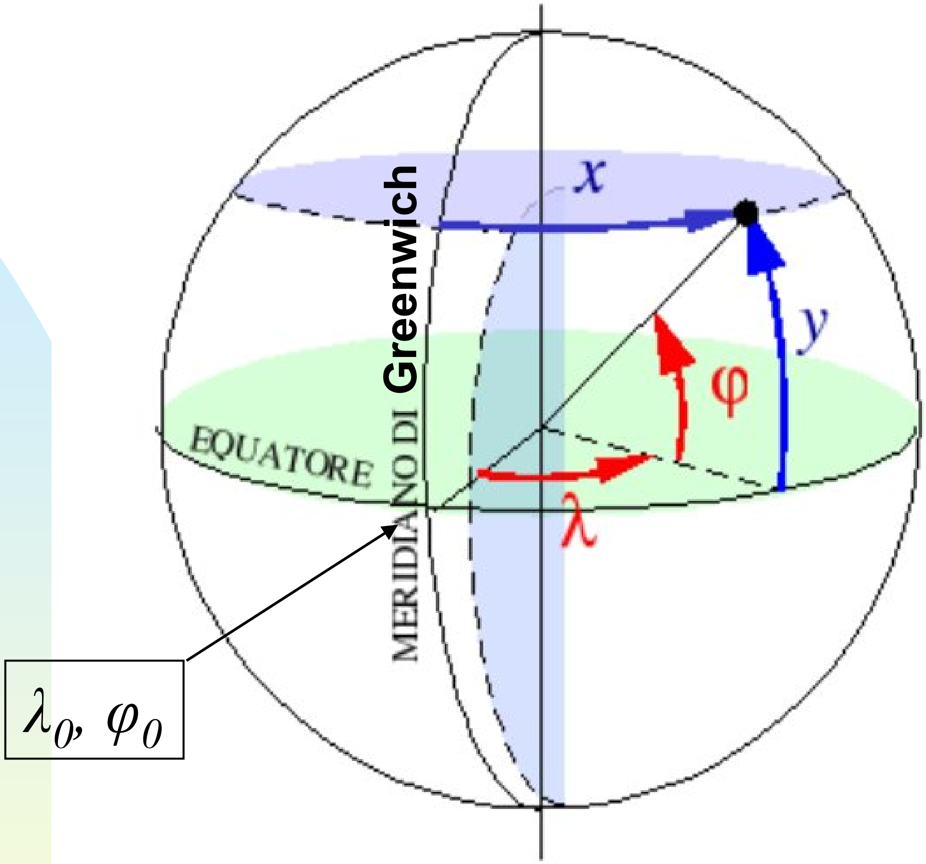
$$\varepsilon = R_d \, / \, R_v$$

Altre variabili prognostiche

$\overline{E}$ Energia cinetica turbolenta

Parametri del suolo:

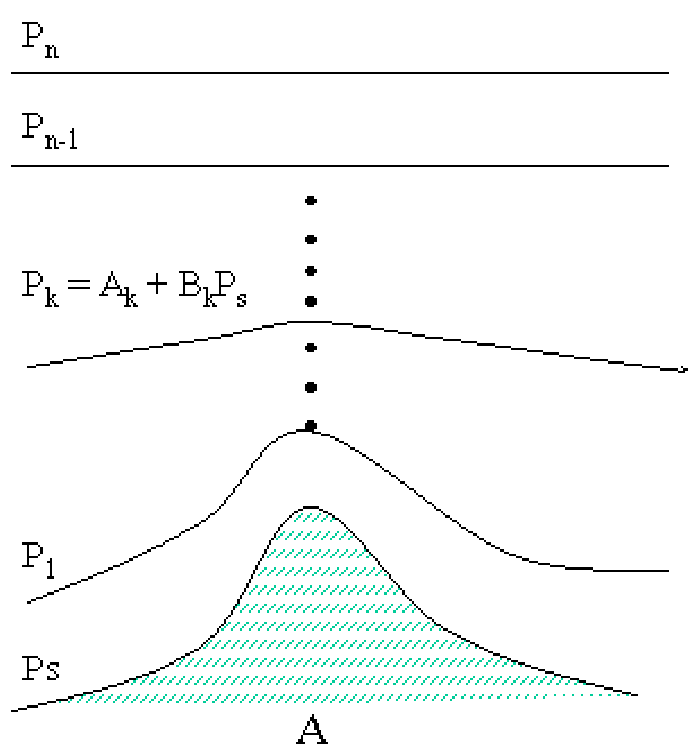
- TG_1, TG_2, TG_3
- QG_1, QG_2, QG_3
- $Hsnow$ (m H_2O)
- $Wveg$ (m H_2O)
- QG_{ICE1}, QG_{ICE2}, QG_{ICE3}

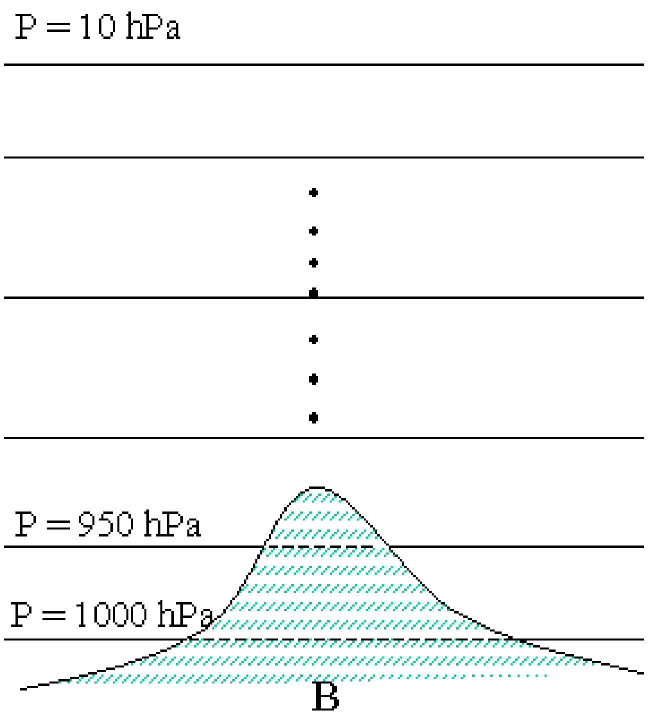


$$x = a \cos(\varphi)\, \lambda$$

$$y = a\, \varphi$$

Coordinata verticale 'terrain following'





Coordinata verticale ‘sigma ibrida’

$$P = P_0\sigma - (P_0 - P_S)\sigma^{\alpha}$$

$$\alpha \leq \frac{P_0}{P_0 - \min(P_S)}$$

$$P = A + BP_S$$

$$A = P_0\sigma - P_0\sigma^{\alpha}$$

$$B = P_0\sigma^{\alpha}$$

$$z \Rightarrow \sigma, \qquad w \Rightarrow \dot{\sigma} = \frac{d\sigma}{dt}$$

Tre cose da sapere per fare la trasformazione di coordinate

1- come si trasforma la derivata totale o Lagrangiana

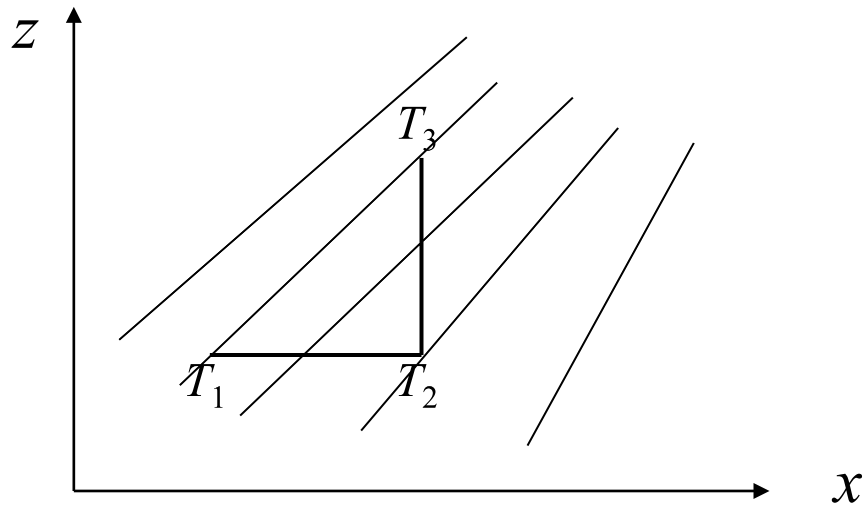
$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$= \frac{\partial}{\partial t} + \frac{u}{a\cos\varphi}\frac{\partial}{\partial \lambda} + \frac{v}{a}\frac{\partial}{\partial \varphi} + \dot{\sigma}\frac{\partial}{\partial \sigma}$$

2- come si trasforma la derivata verticale

$$\left.\frac{\partial}{\partial z}\right|_{x,y,t} = \left.\frac{\partial \sigma}{\partial z}\right|_{x,y,t} \left.\frac{\partial}{\partial \sigma}\right|_{x,y,t}$$

3- come si trasforma la derivata orizzontale



$$\frac{T_3 - T_1}{\Delta x} = \frac{T_2 - T_1}{\Delta x} + \frac{T_3 - T_2}{\Delta x}$$

$$\Rightarrow \quad \left.\frac{\partial}{\partial x}\right|_z = \left.\frac{\partial}{\partial x}\right|_\sigma - \left.\frac{\partial z}{\partial x}\right|_\sigma \frac{\partial}{\partial z}$$

Equazioni del momento in forma analitica in coordinate sferiche

$$\frac{du}{dt} - uv\frac{\tan\varphi}{a} - 2\Omega v\sin\varphi^{*} = K_u - \frac{R_d T_V}{a\cos\varphi}\frac{\sigma^{\alpha}}{P_0\sigma - (P_0 - P_S)\sigma^{\alpha}}\partial_{\lambda}P_S - \frac{1}{a\cos\varphi}\partial_{\lambda}\Phi$$

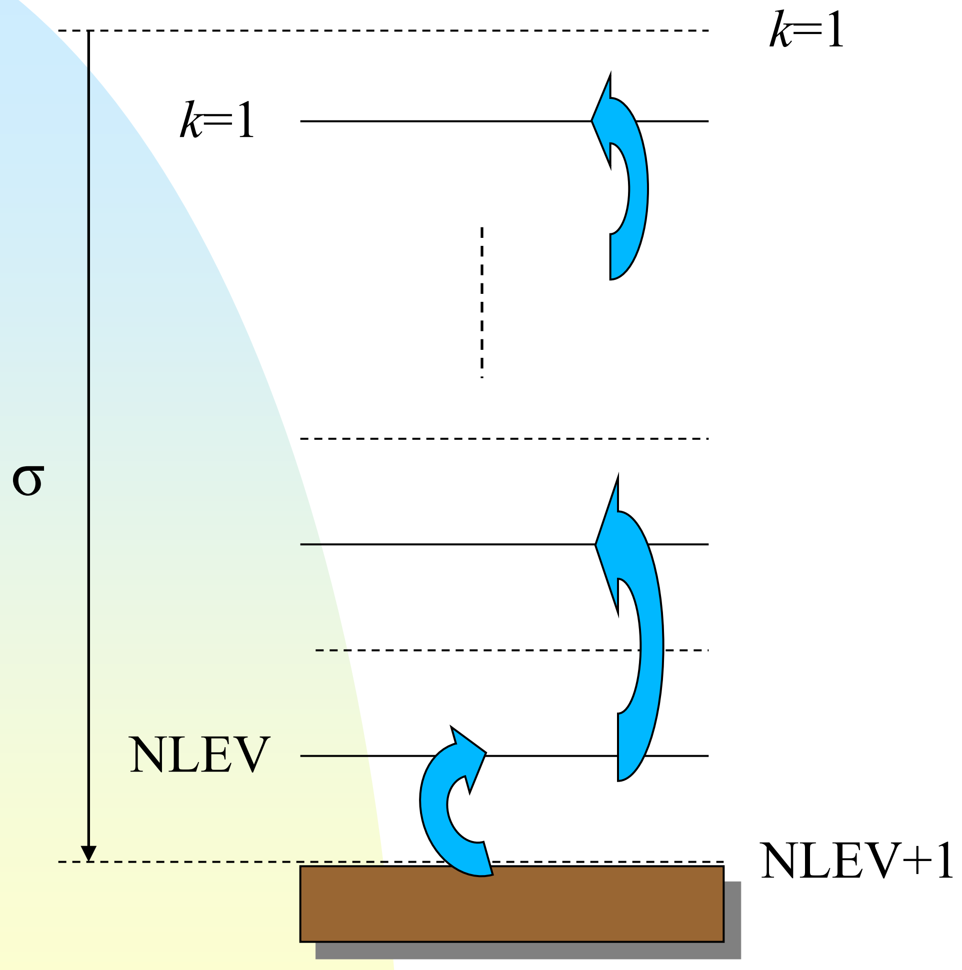
$$\frac{dv}{dt} + u^2\frac{\tan\varphi}{a} + 2\Omega u\sin\varphi^{*} = K_v - \frac{R_d T_V}{a}\frac{\sigma^{\alpha}}{P_0\sigma - (P_0 - P_S)\sigma^{\alpha}}\partial_{\varphi}P_S - \frac{1}{a}\partial_{\varphi}\Phi$$

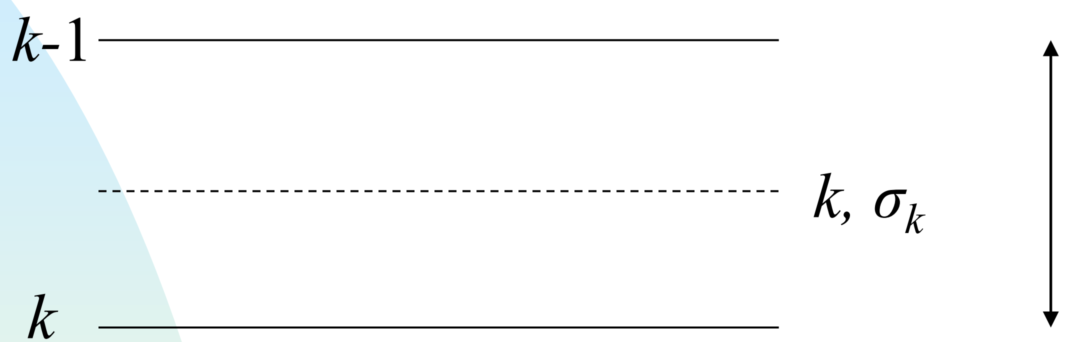
Legge idrostatica

$$\frac{\partial \Phi}{\partial \sigma} = -R_d T_V \frac{P_0 - \alpha (P_0 - P_S)\sigma^{\alpha-1}}{P_0 \sigma - (P_0 - P_S)\sigma^{\alpha}}$$

$\Phi = gz$ geopotenziale

Discretizzazione eq. idrostatica





$\Delta\sigma = \sigma^{\text{int}}_{k} - \sigma^{\text{int}}_{k\text{-}1}$

$$\Phi_{k-1} = \Phi_k + \frac{R_d \overline{T}_V}{\sigma_k} \frac{P_0 - \alpha (P_0 - P_S) \sigma_k^{\alpha - 1}}{P_0 - (P_0 - P_S) \sigma_k^{\alpha - 1}} \Delta\sigma$$

$$\overline{T}_V = .5\,(T_{V\,k} + T_{V\,k-1})$$

Equazione di continuità:

$$\partial_t\left(\frac{\partial P}{\partial\sigma}\right)+\partial_x\left(u\frac{\partial P}{\partial\sigma}\right)+\partial_y\left(v\frac{\partial P}{\partial\sigma}\right)+\partial_\sigma\left(\dot{\sigma}\frac{\partial P}{\partial\sigma}\right) = 0$$

$$\frac{\partial P_S}{\partial t}=-\int_0^1 D\,d\sigma$$

$$D = P_0(1-\alpha\,\sigma^{\alpha-1})D_1+\alpha\,\sigma^{\alpha-1}D_2\left(P_S\right)$$

$$D_1 = \frac{1}{a\cos\theta}\left[\partial_\lambda u+\partial_\theta(v\cos\theta)\right]$$

$$D_2\left(P_S\right) = \frac{1}{a\cos\theta}\left[\partial_\lambda(uP_S)+\partial_\theta(v\,P_S\cos\theta)\right]$$

$$\dot{\sigma}\frac{\partial P}{\partial\sigma}=-\sigma^\alpha\frac{\partial P_S}{\partial t}-\int_0^\sigma D\,d\sigma$$

Velocità verticale generalizzata

Prima legge della termodinamica:

$$\frac{dT_V}{dt} = \frac{R_d T_V \omega}{C_P \ P}$$

$$\omega = \sigma^{\alpha}\left[D_2(P_S) - P_S D_1\right] - \int_0^{\sigma} D d\sigma$$

$$\Rightarrow$$

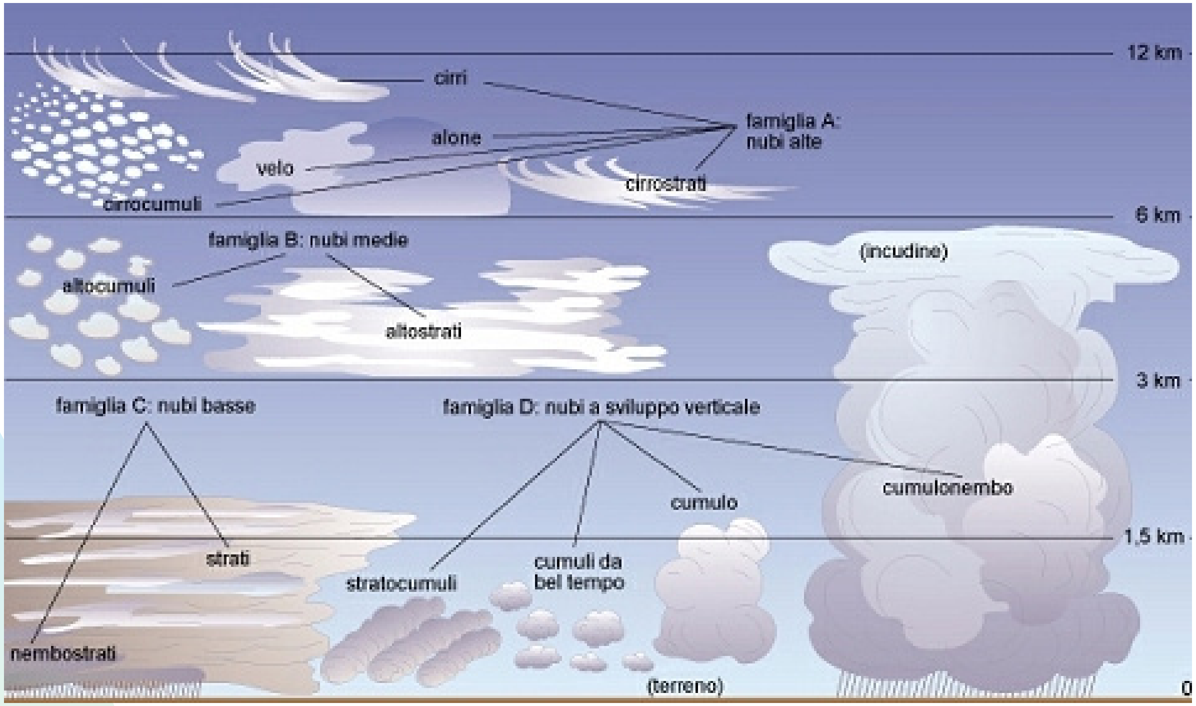
$$\frac{\partial T_V}{\partial t} = -\frac{u}{a\cos\varphi}\frac{\partial T_V}{\partial \lambda} - \frac{v}{a}\frac{\partial T_V}{\partial \varphi} - \dot{\sigma}\frac{\partial T_V}{\partial \sigma} + \frac{R_d T_V \omega}{C_P \ P} + K_T + F_T$$

Equazione di conservazione dell'umidità specifica:

$$\frac{\partial q}{\partial t} = -\frac{u}{a\cos\varphi}\frac{\partial q}{\partial \lambda} - \frac{v}{a}\frac{\partial q}{\partial \varphi} - \dot{\sigma}\frac{\partial q}{\partial \sigma} + K_q + F_q$$

Conservazione delle nubi e idrometeore:

$$\frac{\partial q_k}{\partial t} = -\frac{u}{a\cos\varphi}\frac{\partial q_k}{\partial \lambda} - \frac{v}{a}\frac{\partial q_k}{\partial \varphi} - \dot{\sigma}\frac{\partial q_k}{\partial \sigma} + F_{q_k}$$


12 km
cirri
famiglia A:
nubi alte
alone
velo
cirrostrati
cirrocumuli
6 km
famiglia B: nubi medie
(incudine)
altocumuli
altostrati
3 km
famiglia C: nubi basse
famiglia D: nubi a sviluppo verticale
cumulonembo
cumulo
1,5 km
strati
cumuli da
bel tempo
stratocumuli
nembostrati
(terreno)
0